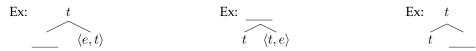
# F13 Type composition exercise

Course in Semantics · Ling 531 / 731 McKenzie · University of Kansas

Given the structure, fill in the missing type. Note that this is an abstract exercise. The structures might not correspond to anything we see. The point is to work with types, whatever they happen to be.



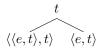
In the first one, the mother is of type t. The right daughter is of type  $\langle e, t \rangle$ , meaning that it is a function that takes something of type e and returns something of type e. So we can conclude that the left daughter is of type e.



The same goes for the other cases. In the second, the function is from t to e; the sister (the input) is type t, so the output (the mother) must be of type e.



Functions can be the input to functions. So the first example could have been different. The mother is of type t, and the right daughter is of type  $\langle e,t\rangle$ . What if the right daughter is the input to the left? What type would that be? Well, its input would be of type  $\langle e,t\rangle$  and its output of type t, so it would have to be  $\langle \langle e,t\rangle,t\rangle$ .



Which answer is right, then? Either one is. Sometimes we'll want one, sometimes the other.

### 1 Figuring out the output type

In each instance, figure out the type of the result of combining the two listed nodes by functional application.

### 1.1 simple type argument

1.

2.  $\langle e, t \rangle = e$ 

3.  $\langle s, e \rangle$  s

 $\langle e, e \rangle$ 

5.  $\overbrace{t \quad \langle t, t \rangle}$ 

6.  $s = \langle s, t \rangle$ 

7.  $\overbrace{i \quad \langle i, e \rangle}$ 

8.  $e^{-\langle t, e \rangle}$ 

### 1.2 function arguments (i.e. functions as arguments)

1.  $\overbrace{\langle\langle e,t\rangle,t\rangle\quad\langle e,t\rangle}$ 

2.  $\langle \langle s, e \rangle, t \rangle \quad \langle s, e \rangle$ 

3.  $(\langle e, t \rangle, e \rangle \quad \langle e, t \rangle$ 

4.  $\langle e, t \rangle \quad \langle \langle e, t \rangle, i \rangle$ 

5.  $\langle l, t \rangle \quad \langle \langle l, t \rangle, l \rangle$ 

6.  $\overbrace{\langle t,t\rangle \quad \langle \langle t,t\rangle,t\rangle}$ 

7.  $\langle e, t \rangle \quad \langle \langle e, t \rangle, \langle s, t \rangle \rangle$ 

8.  $\langle l,t \rangle$   $\langle \langle l,t \rangle, \langle e, \langle s,t \rangle \rangle \rangle$ 

). 
$$\langle e,t \rangle$$
  $\langle \langle e,t \rangle, \langle e, \langle e,t \rangle \rangle \rangle$ 

## 2 Figuring out input types

In each instance, figure out the type of one of the inputs by deducing from the listed nodes.

### 2.1 Simple-type argument

All the answers here are of simple type, even if complex types are technically possible.

1. t  $\langle e, t \rangle$ 

2.  $\langle e, t \rangle$   $\langle e, \langle e, t \rangle \rangle$ 

3.  $\underbrace{e}{\langle t, e \rangle}$ 

4. e  $\langle e, e \rangle$ 

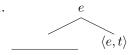
5.  $\langle s, t \rangle$ 

6.  $\langle i, \langle s, t \rangle \rangle$ 

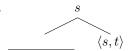
### 2.2 Complex-type argument

All of the answers here are of complex type ( $\langle \sigma, \tau \rangle$ ), even if simple types are technically possible.

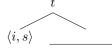
1.



2.

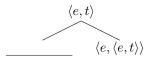


3.

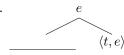


4. t

5.

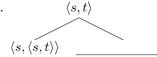


6.

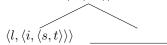


7. e  $\langle e, e \rangle$ 

8.



9.



### 3 Larger structure

Give this one a go.

