Q03 Universal quantification

Course in Semantics · Ling 531 / 731

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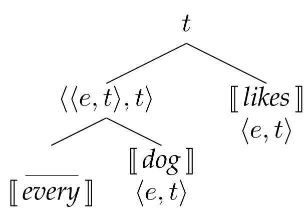
1 at 1:00 - The type of a quantifier

The notion of quantifiers (over entities) was worked out without considering event arguments, so we'll see those first.

(1) *Every [dog]$_{NP}$ [likes bacon]$_{VP}$.*
= 1 iff every dog likes bacon

(2) a. $[\![\textit{dog}]\!] = \lambda x \in D_e.\ \text{dog}(x) : \langle e, t\rangle$
b. $[\![\textit{likes}]\!] = \lambda y \in D_e.\ \text{likes(bacon)}(y) : \langle e, t\rangle$

Putting these together, we can see that $[\![\textit{every dog}]\!]$ is of type e or of type $\langle et, t\rangle$. Type e is unlikely because quantifiers do not refer. So the quantifier phrase is of type $\langle\langle e, t\rangle, t\rangle$, a set of properties. What does that make the type of the determiner *every*?



That's right, it's $\langle\langle e, t\rangle, \langle\langle e, t\rangle, t\rangle\rangle$. We usually write this as $\langle et, \langle et, t\rangle\rangle$. Whenever you see any type of $\langle \alpha, \langle \beta, t\rangle\rangle$, you are seeing a **relation** between objects of type α and objects of type β. So an intransitive verb of type $\langle e, st\rangle$ is a relation between entities and events. When the objects are of the same type, like $\langle \alpha, \langle \alpha, t\rangle\rangle$, we have a relation between (two) objects of the same type. Thus, a determiner like *every* denotes a relation between properties of entities.

In the case of the universal quantifier, the relation is this: Every object with the first property also has the other.

2 at 5:50 - Relating two properties

Every object with the first property also has the other = Every object in the first set is also in the second set.

The truth conditions of a sentence with a universal quantifier thus depends on checking every object in the first set to see if it's in the second.[1] Does Fido like bacon? Does Odie? Does Snoopy?, and so on.

[1]Obviously, we can't check *every* single dog on Earth. So there is a facet of contextual domain restriction in the use of universal quantifiers. We'll set that aside for now.

That means it's easy to create a context for these. Positive contexts lead to truth if the VP holds of every member of the set, and negative contexts lead to falsehood if the VP fails to hold of even one member.

But we can**not** conjoin the two sets, as in (3).

(3) every $x \in D_e \mid x \in [\![\textit{dog}]\!] \;\&\; x \in [\![\textit{likes bacon}]\!]$

Conjunctions are false if *either* conjunct is false, so *Every dog likes bacon* would lead to falsehood if there is an entity that is not a dog. But crucially, non-dogs have **no bearing** on the truth of (1).

We only want the second conjunct link properties with an *if*. Material implication, the philosophers call it. Every object is such that IF it is in the first set it is also in the second. (1) can be recast: Every object is such that IF it is a dog it also likes bacon.

(4) every $x \in D_e \mid x \in [\![\textit{dog}]\!] \rightarrow x \in [\![\textit{likes bacon}]\!]$

(5) $\forall x \in D_e$ [dog(x) = 1 $\rightarrow$ likes(bacon)(x) = 1]

3 at 11:15 - Unpacking to find $[\![\textit{every}]\!]$

$[\![\textit{every}]\!]$ denotes a relation between two properties. Therefore, it also takes two function arguments. We can replace the functions in the value condition with variables; whatever the NP is will be plugged in here.

(6) a. $\forall x \in D_e$ [dog (x) = 1 $\rightarrow$ likes(bacon) (x) = 1]

b. $\forall x \in D_e$ [f (x) = 1 $\rightarrow$ g (x) = 1]

c. $\lambda f \in D_{e,t}$. $\lambda g \in D_{e,t}$. $\forall x \in D_e$ [f (x) = 1 $\rightarrow$ g (x) = 1]